Simplification Study on Dynamic Models of Distributed Parameter Systems

Mansour Shirvani
Chemical Engineering, Iran University of Science and Technology, 16844, Tehran, Iran

Makoto Inagaki and Tadaaki Shimizu
Dept. of Material and Chemical Engineering, Niigata University, Niigata 950-21, Japan

The dynamic model of many distributed parameter systems can be approximated by a quasi-rational transfer function (Ramanathan, 1988, 1989; Jerome and Ray, 1991). Such a transfer function model is the parallel connection of two rational transfer functions, including time delay. Quasi-rational transfer functions can describe the dynamic behavior of many processes in which hyperbolic partial differential equations describe the mathematical model of the system. A quasi-rational transfer function is of the form:

$$G(s)=\frac{P_a(s)+P_b(s)e^{-\tau_d s}}{Q(s)} \tag{1}$$

in which $P_a(s)$, $P_b(s)$ and $Q(s)$ are polynomials in s. Resonance behavior in the frequency response of these systems, as well as the existence of minimum and nonminimum phase behavior, are the specific features known for them. The occasional appearance of a break point in the step response is another specific characteristic of quasi-rational transfer function models.

The roots of a quasi-rational model, that is, the zeros of Eq. 1, are important, whether they are located in the right half or in the left half of the complex plane. According to Bellman and Cooke (1963) and Ramanathan (1988, 1989), the location of zeros is predictable by comparison of the orders of the polynomials $P_a(s)$ and $P_b(s)$ at high frequencies, if they are not equal in order. If $P_a(s)$ and $P_b(s)$ are equal in order, then the location of zeros is determined according to the relative value of the coefficients of the highest order of s in $P_a(s)$ and $P_b(s)$. On the other hand, searching for simplified models in process control is an important issue. The problem becomes more complicated in distributed parameter systems compared to lumped parameter systems, due to the transcendental characteristics of these systems.

Simplified Model

The simplified model of quasi-rational systems may be put in the form:

$$G(s)=g_1(s)+g_2(s), \tag{2}$$

where

$$g_1(s)=\frac{K_1P_1(s)}{Q_1(s)} \quad \text{and} \quad g_2(s)=\frac{K_2P_2(s)e^{-\tau_d s}}{Q_2(s)}.$$

$P_1(s)$, $P_2(s)$, $Q_1(s)$ and $Q_2(s)$ are polynomials in s with their zero-order terms equal to 1. Actually the model 2 is of the same form as 1, except that the denominators $Q(s)$ may or may not be equal for both terms in the model. The same conditional remarks regarding the appearance of right half or left half plane zeros (appearance of minimum or nonminimum phase characteristics) are valid also for the model 2, except that one should consider the relative orders of $P_1(s)/Q_1(s)$ and $P_2(s)/Q_2(s)$ in place of the orders of $P_a(s)$ and $P_b(s)$, respectively. Thus, the model has an infinite number of right half plane zeros and exhibits nonminimum phase characteristics if:

$$\left[\text{Relative order of } \frac{P_2(s)}{Q_2(s)}\right] > \left[\text{Relative order of } \frac{P_1(s)}{Q_1(s)}\right]$$

or

$$\left[\text{Relative order of } \frac{P_2(s)}{Q_2(s)}\right] = \left[\text{Relative order of } \frac{P_1(s)}{Q_1(s)}\right]$$

and

$$\frac{K_2 c_{p2}}{c_{q2}} > \frac{K_1 c_{p1}}{c_{q1}}.$$

Conversely the model is minimum phase if:

$$\left[\text{Relative order of } \frac{P_2(s)}{Q_2(s)}\right] < \left[\text{Relative order of } \frac{P_1(s)}{Q_1(s)}\right]$$

or

$$\left[\text{Relative order of } \frac{P_2(s)}{Q_2(s)}\right] = \left[\text{Relative order of } \frac{P_1(s)}{Q_1(s)}\right]$$

and

$$\frac{K_2 c_{p2}}{c_{q2}} \leq \frac{K_1 c_{p1}}{c_{q1}}$$

provided that $g_1(s)$ is a minimum phase rational transfer function, that is, no right half plane zeros appear in $g_1(s)$. c_{p1}, c_{p2}, c_{q1}, and c_{q2} are the coefficients of highest order of s in the polynomials $P_1(s)$, $P_2(s)$, $Q_1(s)$ and $Q_2(s)$, respectively.

The inherent capability of both the high-order quasi-rational model 1 and the simplified model 2 to exhibit minimum, as well as nonminimum phase characteristics, supports the use of model 2 for approximating quasi-rational systems 1. Furthermore, as will be seen in the simulation example, the simplified model can also exhibit resonance behavior, which has been observed in quasi-rational systems.

Simulation

An example is presented in this section in which the simplified model 2 is simulated and compared to a complicated quasi-rational system. The above concepts were used for predicting a suitable order for the polynomials of the simplified model. We select a simplified model in the form

$$\frac{K_1(\tau_{n1}s+1)e^{-\tau_{d1}s}}{\tau_1^2 s^2 + 2\zeta_1 \tau_1 s + 1} + \frac{K_2 e^{-\tau_{d2}s}}{\tau_2 s + 1} \tag{3}$$

for fitting the frequency data of a MSMPR crystallizer system. The theoretical procedure for deriving the detailed transfer function model of this system was presented by Ramanathan (1988).

The simulation was performed in frequency domain, and the result of simulation of the simplified model and that of the original system are compared in Figure 1.

The procedure for selecting the orders of the individual polynomials of the model 3 is as follows:

(1) The data show a peak in the first maximum point appearing in the gain diagram. This suggests using a second-order $Q(s)$ in one of the elements of the model. The $Q(s)$ for the other element may be selected as first-order, since there is no reason to increase its order. Here, $Q_1(s)$ was selected as second-order and $Q_2(s)$ as first-order polynomials in s.

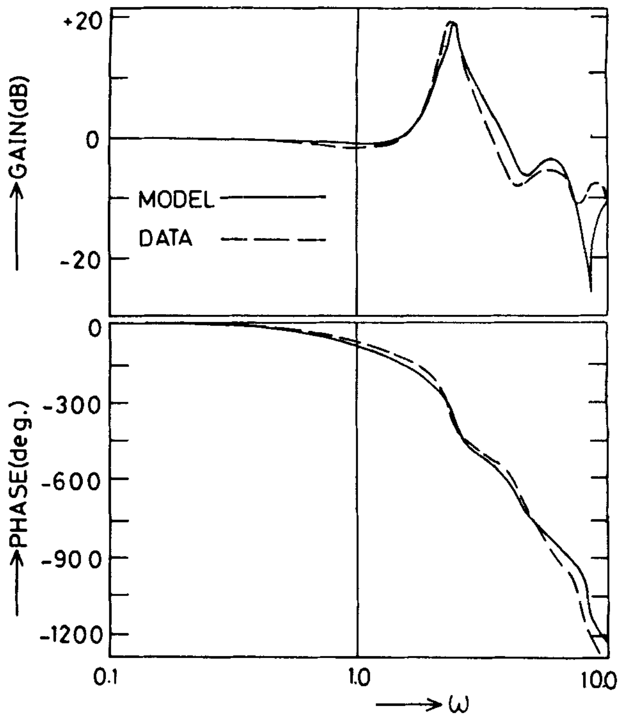

Figure 1. Simplified model fitted to the frequency response data of a crystallizer.

(2) The average slope of the gain of the data is seen to be around -20dB/dec. This means that the relative order of $P_1(s)/Q_1(s)$ and $P_2(s)/Q_2(s)$ both should be equal to -1. As a result, the order of $P_1(s)$ should be selected equal to one for compensating the excess order of $Q_1(s)$.

(3) The phase angle of the data indicate a nonminimum phase system. If the system were a minimum phase one, then, certainly one of the time delay parameters (for example, τ_{d1}) should become equal to zero and the corresponding $P_1(s)$ should have no right half plane root. For this example, the nonminimum phase behavior of the system implies that:

$$\frac{K_1 \tau_{n1}}{\tau_1^2} > \frac{K_2}{\tau_2} \quad \text{if} \quad \tau_{d1} > \tau_{d2}$$

or

$$\frac{K_1 \tau_{n1}}{\tau_1^2} < \frac{K_2}{\tau_2} \quad \text{if} \quad \tau_{d1} < \tau_{d2}.$$

This conclusion can be used to check the results of calculation of parameters.

In addition to the above observations, from the gain of the data at low frequencies, it is seen that $K_1 + K_2 = 1$. Thus, $K_2 = 1 - K_1$ can be calculated directly, so that the model and data become completely fitted at initial frequencies.

The method of simulation used here for calculating the values of the parameters was based on minimization of the length of the difference vector $\Delta G(j\omega)$ between the frequency response data and the model, as is shown in Figure 2.

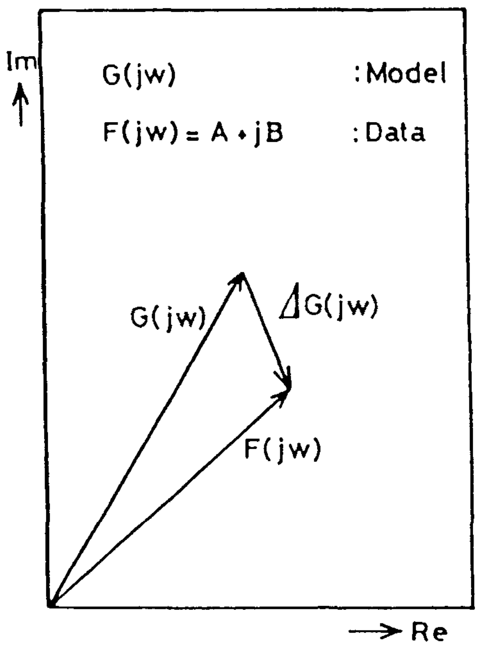

Figure 2. Vector difference concept for fitting the simplified model and data of a crystallizer.

The method of steepest descent was used for the minimization. The calculated values of the parameters were:

$K_1 = 0.68$	$\tau_1 = 0.43$	$\tau_{n1} = 0.43$	$\zeta_1 = 0.05$	$\tau_{d1} = 1.5$
$K_2 = 0.32$	$\tau_2 = 0.20$	—	—	$\tau_{d2} = 0$

The step responses of the simplified model and the original model of the system are compared in Figure 3. The break point in the step response, which appears exactly at $t = \tau_{d1}$, is well recognizable in this model.

Discussion

The method of minimizing the length of the difference vector $\Delta G(j\omega)$ enables one to calculate the values of the parameters of the model by fitting both the gain and phase angle simultaneously. The choice of suitable orders for the individual polynomials of the model were based on the qualitative characteristics of the frequency response data.

After determining the orders of the polynomials of the simplified model, it is possible to perform the calculation of the values of the parameters based on step response data instead of frequency response data. In such a case, the common method of least-squares curve fitting will result in a nonlinear regression and calculation of the values of the parameters of the simplified model. However, contrary to frequency response data, the form and appearance of step response data cannot help in predicting the most suitable order for the polynomials.

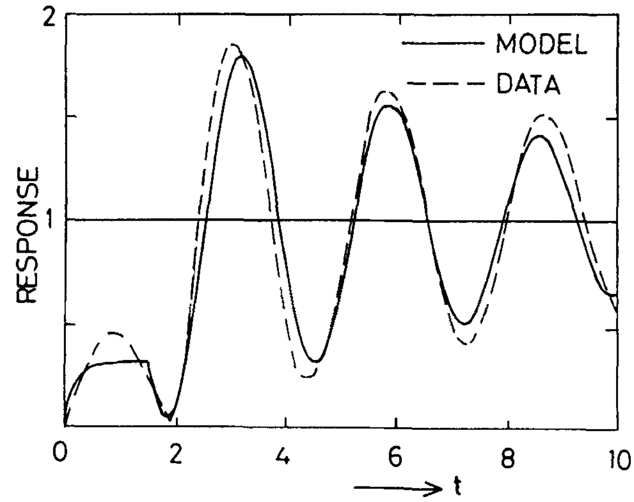

Figure 3. Step response of the simplified model vs. the original system.

The general form of the simplified model 2 is useful in approximating general distributed parameter systems that are governed by hyperbolic partial differential equations and quasi-rational transfer function models.

Examples of such systems are heat exchangers, packed towers, particulate systems, like crystallizers and fluidized-bed calciners. The above method of simulation can be used when the frequency response data of a system that falls in this category are available. For all of the systems, resonance behavior appear and the slope of gain at high frequencies can be used as a guide for predicting the order of the polynomials.

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